Electromagnetic simulation of rotating propeller blades for radar detection purposes

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An integral equation method is used for the modeling of electromagnetic scattering from a rotating propeller blade, of which the length is about half a wavelength. This provides a convenient way of calculating its backscattering spectrum that is useful, e.g., when radar detection capabilities are studied. The theoretical model is able to predict the discrete nature of the scattered spectrum and the asymmetry with respect to the base frequency of the incident wave. The simulations are compared with anechoic chamber measurements for different orientations of the propeller's axis with respect to the polarization of the incident wave.

***Index Terms*—integral equation, micro-Doppler, propeller blade, radar detection, wave scattering**

I. Introduction

RECENTLY, Unmanned Aerial Vehicles (UAVs) — drones — have gotten to the focus of attention: they are widespread, however, there are difficulties in detecting them with conventional radar systems due to their very small radar cross section (RCS). In order to develop more powerful detection methods, it is necessary to more deeply investigate the patterns in the signal reflected from UAVs [1].

Drone propeller blades are commonly made of Carbon-Fiber Reinforced Plastic; for scattering problems at wavelengths in the order of magnitude of its length ($l \approx 25\,\mathrm{cm} \approx \lambda$; $f \approx 100\,\mathrm{MHz} - 1\,\mathrm{GHz}$), it can be considered to be a conductor [2]; this has been confirmed experimentally as well by replacing the propeller blade with a cuboid of a similar shape coated with copper during the anechoic chamber measurements. If such a rotor blade is investigated using electromagnetic waves at its resonant frequency ($\lambda \approx \frac{l}{2}$), one can intuitively expect a maximal amount of current induced and correspondingly, a strong scattered field. In addition, the RCS is strongly dependent on the orientation of the rotor blade [1]. Also Digital Video Broadcasting - Terrestrial (DVB-T) transmitters operate in this frequency band, giving rise to passive radar systems for drone detection purposes.

Because of the rotation of the propeller blades, the scattered field will have a certain pattern corresponding to the modulation caused by this rotation (the micro-Doppler effect). A physical optics model [3], [4] is often used to compute the scattered field; however, this method fails in the case of the short propeller with length $l \approx 2\lambda$. The approach presented herein is governed by the full set of Maxwell's equations, taking into account the rotation of the observed object.

II. Computational models

Scattering by conductors can be modelled by, e.g., the Method of Moments (MoM) [5]. The MoM is computationally efficient in the case of thin propeller blades because even a 1-dimensional discretization is appropriate as only the longitudinal component of the induced current is significant. Consequently, a much less number of degrees of freedom is introduced compared to other numerical methods based on a volume discretization, e.g., the finite element method.

In the first step, we calculate currents induced by the incident EM wave. The current is approximated as a weighted sum of basis functions. The use of global basis functions is advantageous in some cases [5]. For non-closed thin wires, there is a requirement of zero current at the ends of the wire. We use a sequence of harmonic functions satisfying this condition in the form of

$$f_n(r) = \sqrt{\frac{2}{l}} \sin\left(n\frac{\pi r}{l}\right), \quad n = 1, 2, 3, \ldots \tag{1}$$

where r is the position on the propeller with length l such that $r \in [-l/2, l/2]$. Once the induced currents are calculated, the scattered electric field can be determined. The full RCS is then calculated from the scattered electric far field as:

$$\sigma = \lim_{\rho\to\infty} 4\pi\rho^2 \frac{|\mathbf{E}_s|^2}{|\mathbf{E}_i|^2}, \tag{2}$$

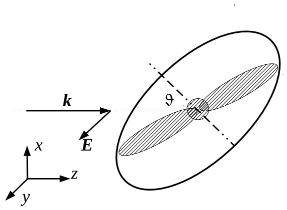
where $\mathbf{E}_s$ is the scattered electric field in the direction of observation at a distance ρ and $\mathbf{E}_i$ is the incident electric field. The polarimetric RCS can be calculated by taking a certain (for example x) component of the scattered field.

III. Scattering from a rotating propeller blade

In this section, we account for the modulation of the scattered wave caused by the rotation of the scatterer.

The approach used herein is to transform the incident field to corotating coordinates, calculate the scattered currents to compute the scattered field in the corotating coordinates, then transform back to the laboratory coordinates (for a rigorous approach, see for example [6]; at the low velocities of this problem ($v \leq 100\,\mathrm{m\,s^{-1}}$), relativistic effects need not be considered).

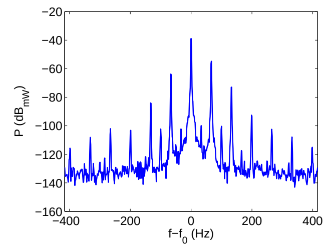
In the setup shown in Fig. 1a, the axis of rotation (and so the angular velocity vector) is perpendicular to the y axis ($\mathbf{\Omega} \perp \hat{\mathbf{y}}$). When calculating the induced currents, only the component of



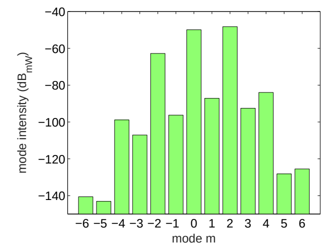
(a) Coordinate system

(b) Experimental setup

Fig. 1. The studied configuration



(a) measured spectrum for $\vartheta = 45^o$



(b) Calculated intensities corresponding to angular frequency $\omega + m\Omega$

Fig. 2. Comparison of measured and calculated spectra for a circularly polarized incident wave ($\Omega = 2\pi \times 33.3\,\mathrm{s}^{-1}$). The large difference for $m = 0$ is caused by miscellaneous scattering from the static environment.

the electric field parallel with the propeller's length is relevant. In corotating coordinates, this can be expressed as:

$$E_r = E_y \sin(\Omega t) \exp(\mathrm{j}\omega t) \exp(-\mathrm{j}kr\sin(\vartheta)\cos(\Omega t)), \quad (3)$$

where ω and k are the angular frequency and the wavenumber of the EM wave, Ω is the angular frequency of the rotation. The term in the second exponential arises as a consequence of phase delay of the incident field. Using the fact that:

$$\exp(-\mathrm{j}kr\cos(\Omega t)) = \sum_{m=-\infty}^{\infty} \exp\left(\mathrm{j}m\left(\Omega t - \frac{\pi}{2}\right)\right) J_m(kr\sin(\vartheta)), \quad (4)$$

where J_m is the m-th order Bessel's function of the first kind, and:

$$J_{m-1}(x) + J_{m+1}(x) = \frac{2m}{x} J_m(x), \quad (5)$$

we can express the incident radial electric field as:

$$E_r = E_y \sum_{m=-\infty}^{\infty} \left[\frac{m}{kr\sin(\vartheta)} J_m(kr\sin(\vartheta)) \times \exp\left(-\mathrm{j}m\frac{\pi}{2}\right) \exp(\mathrm{j}(\omega + m\Omega)t)\right]. \quad (6)$$

This means that in a system of reference corotating with the propeller blade, the plane wave can be expressed as a series of electric fields of discrete frequencies. For a half wavelength antenna ($kr \leq \pi/2$), coefficients rapidly decrease with an increasing m, so that for $|m| > 10$ they can be neglected.

The spectrum of currents induced by this discrete spectrum will contain these exact frequencies, as will the scattered fields radiated by these currents. The inverse transformation back into laboratory coordinates results in a discrete spectrum with the same frequencies [7], so the resulting scattered field can be expressed as:

$$\mathbf{E}_s = \sum_{m=-\infty}^{\infty} \mathbf{E}_m \exp(\mathrm{j}(\omega + m\Omega)t). \quad (7)$$

This is consistent with experimental results shown in Fig. 2.

For a rotor blade, we can safely assume that $\Omega \ll \omega$, ($\Omega \sim 10^2\,\mathrm{s}^{-1}$, while, as mentioned earlier, $\omega \sim 10^8\,\mathrm{s}^{-1}$). The currents induced by almost identical frequencies (because $|m\Omega/\omega| \ll 1$ for the considered small values of m in most practical considerations) have a virtually identical distribution pattern along the wire (as coefficients for higher harmonics rapidly disappear). This means that there exists only a weak coupling between rotation and scattering phenomena; the problem can be treated in a quasi-static approximation — calculating the scattered field for positions of the propeller and interpreting the results as a sequence in time, then transforming to frequency domain if needed.

A description of the anechoic chamber experimental setup will be detailed in the full paper, herein we only refer to [1]. This scheme is extended by the capability of varying the orientation of the blade axis as shown in Fig. 1b.

IV. Conclusion, perspectives

Scattering from a rotating propeller blade near its resonant frequency is investigated for different orientations using an integral equation model (MoM). It is shown that the backscattered spectrum consists of discrete frequencies, which is then compared to experimental results. The approach can facilitate either the measurement of a low RCS or the detection of such objects, e.g., UAV propellers. More details on the model, the numerical implementation and its validation will be included in the full version. Moreover, we extend this study for the practically relevant case of multiple propellers rotating in the near field of each other.

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